RECENT IDEAS ON THE FLAVOR PUZZLE

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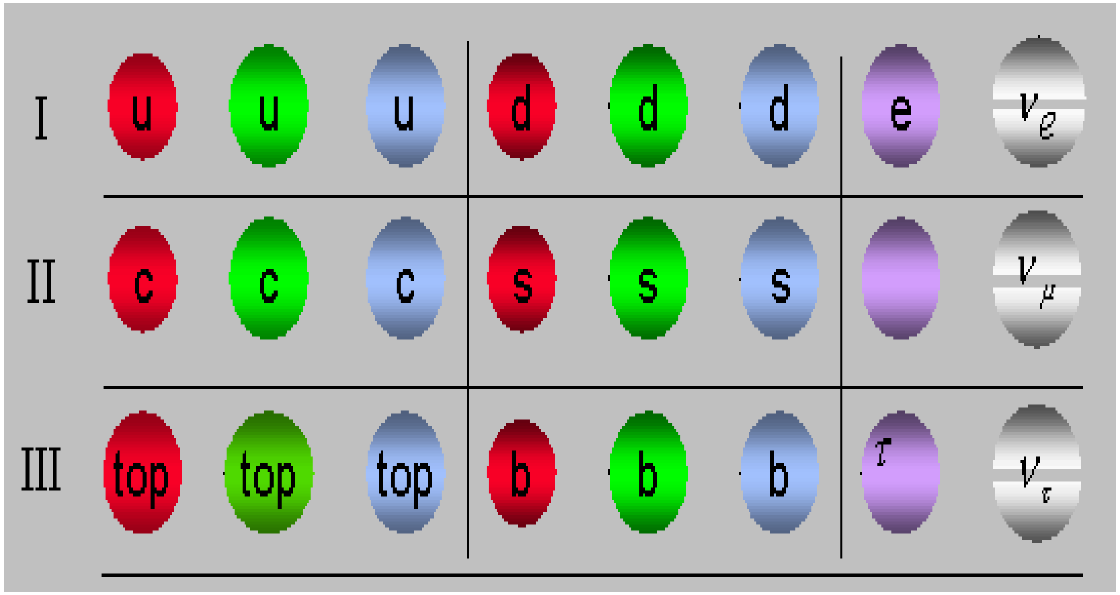
I
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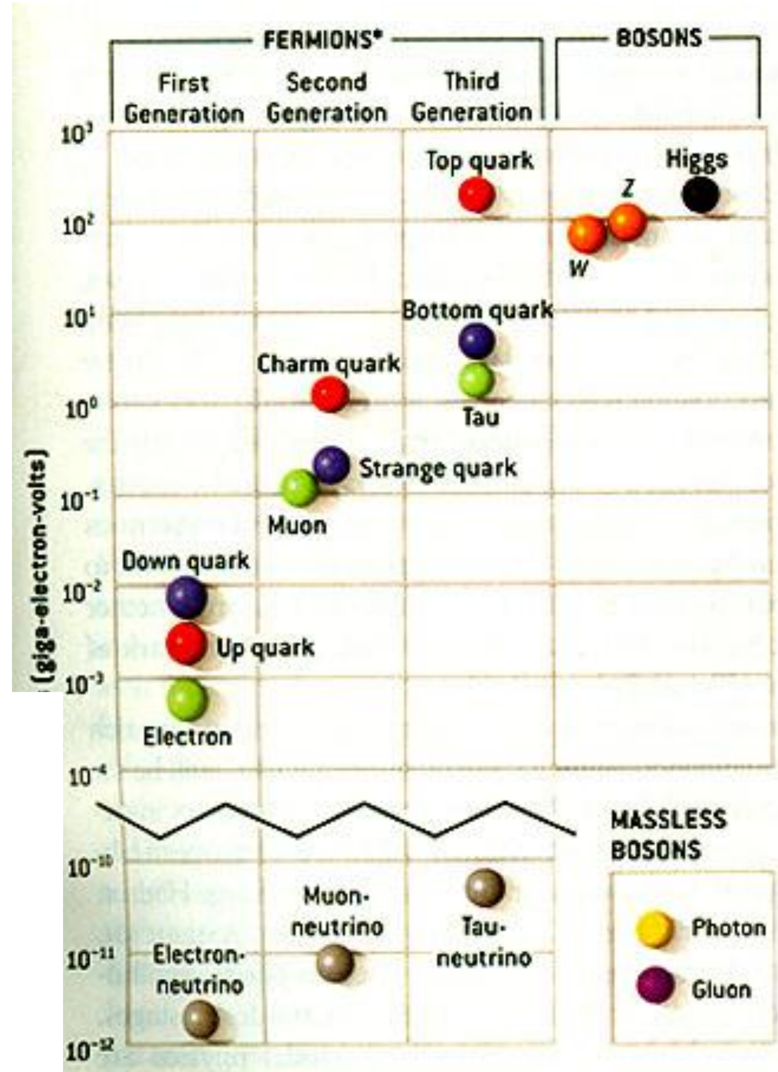
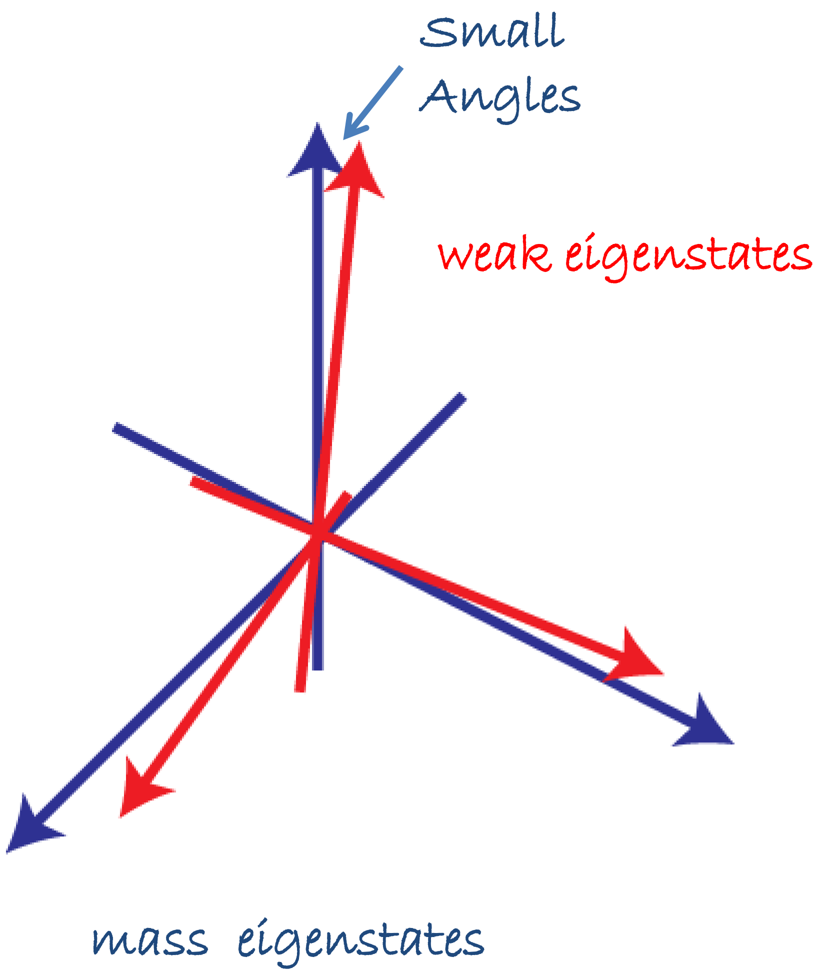
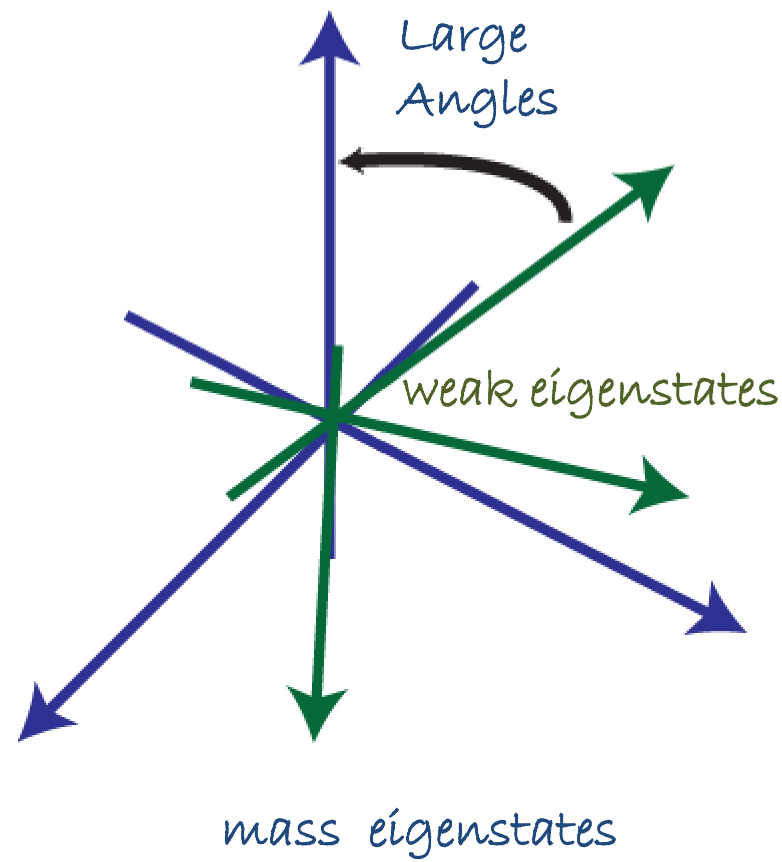

FERMIONS*
BOSONS
First Generation
Second Generation
Third Generation
(giga-electron-volts)
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-10}
10^{-11}
10^{-12}
Top quark
Higgs
Z
W
Bottom quark
Charm quark
Tau
Strange quark
Muon
Down quark
Up quark
Electron
MASSLESS BOSONS
Muon-neutrino
Tau-neutrino
Electron-neutrino
Photon
Gluon

MIXING

QUARKS



LEPTONS



FLAVOR PUZZLE

- **Explain the three-family structure**
- **Pattern of masses and mixings predicted within the theory**

Are the measured masses and mixings hinting us something about the theory behind the 3 generations?

Normal physicists worried about strings and other regular stuff

Physicist that thought too much about the Flavour Puzzle

HOW TO ATTACK THE FLAVOR PUZZLE??

- **SYMMETRY is suggested by family structure**

CONTINUOUS

Minimal Flavor Violation,
Yukawas as scalar fields,
Minimization leads to predictions

DISCRETE

Decoupling of masses and mixings,
Model independent predictions

R. Alonso, B. Gavela, C. S. Fong, G. Isidori, L. Maiani, L. Merlo, E. Nardi, S. Rigolin, D. H.
1103.2915, 1206.3167, 1306.5922, 1306.5927

A. Yu. Smirnov, D. H.

1204.04045, 1212.2149, 1304.7738

THE CONTINUOUS PATH

R. Alonso, C. S. Fong, B. Gavela, G. Isidori, L. Maiani, L. Merlo, E. Nardi, S. Rigolin, D. H.

1103.2915, 1206.3167, 1306.5922, 1306.5927

CONTINUOUS FLAVOR SYMMETRIES

invariant

noninvariant

$$\mathscr{L}_{SM}^{ren} = \mathscr{L}_{kin} - V(\mathrm{Higgs})$$

$$G = SU(3)_{Q_L} \times SU(3)_u \times SU(3)_d \times SU(3)_L \times SU(3)_e$$

spurion

$$Q_u \to L_u Q_u \, , \quad d_R \to R_d d_R \, , \ldots$$

TO RECOVER INVARIANCE!

Georgi, Chivukula; Phys.Lett.B188:99,1987
G. D'Ambrosio, G. Giuduce, G. Isidori, A. Strumia; hep-ph/0207036

WHY SPURIONS?

Hierarchy Problem → *New Physics at the* ***TeV***

VS

Precision Flavor Experiments → *No New Flavor Physics up to 1000 TeV!*

Operator	Bounds on Λ in TeV ($c_{ij}=1$)		Bounds on c_{ij} ($\Lambda = 1$ TeV)		Observables
	Re	Im	Re	Im	
$(\bar{s}_L \gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar{s}_R\, d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	Δm_K; ϵ_K
$(\bar{c}_L \gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p \rvert, \phi_D$
$(\bar{c}_R\, u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	Δm_D; $\lvert q/p \rvert, \phi_D$
$(\bar{b}_L \gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_R\, d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_L \gamma^\mu s_L)^2$	1.1×10^2		7.6×10^{-5}		Δm_{B_s}
$(\bar{b}_R\, s_L)(\bar{b}_L s_R)$	3.7×10^2		1.3×10^{-5}		Δm_{B_s}

G. Isidori, Y. Nir, G. Perez, 1002.0900

MINIMAL FLAVOR VIOLATION (MFV)

$$\mathscr{L} = \mathscr{L}(Y_u, Y_d, \cdots + \text{fields})$$

formally invariant
under flavor!

MFV: *Not only renormalizable but also nonrenormalizable couplings should be invariant under:*

$$Q_u \to L_u Q_u\,, \quad d_R \to R_d d_R\,, \ldots \; Y_d \to L_u Y_d R_d^\dagger$$

HENCE, AT LOW ENERGIES

$$\mathscr{L}_{d\geq 5} = \sum \frac{c^i_{d=6}}{\Lambda^2_{fl}} \mathcal{O}^i_{d=6} + \ldots$$

$$c^i_{d=6} \equiv c^i_{d=6}(Y_u, Y_d)\,, \quad c^j_{d=6} \equiv c^j_{d=6}(Y_u, Y_d)$$

H. Georgi, R. Chivukula; Phys.Lett.B188:99,1987
G. D'Ambrosio, G. Giuduce, G. Isidori, A. Strumia; hep-ph/0207036

also formally invariant!

Minimally flavour violating dimension six operator	main observables	Λ [TeV] −	+	
$\mathcal{O}_0 = \frac{1}{2}(\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)^2$	$\epsilon_K, \quad \Delta m_{B_d}$	6.4	5.0	
$\mathcal{O}_{F1} = H^\dagger \left(\bar{D}_R \lambda_d \lambda_{\rm FC} \sigma_{\mu\nu} Q_L\right) F_{\mu\nu}$	$B \to X_s \gamma$	9.3	12.4	
$\mathcal{O}_{G1} = H^\dagger \left(\bar{D}_R \lambda_d \lambda_{\rm FC} \sigma_{\mu\nu} T^a Q_L\right) G^a_{\mu\nu}$	$B \to X_s \gamma$	2.6	3.5	
$\mathcal{O}_{\ell 1} = (\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)(\bar{L}_L \gamma_\mu L_L)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.1	2.7	*
$\mathcal{O}_{\ell 2} = (\bar{Q}_L \lambda_{\rm FC} \gamma_\mu \tau^a Q_L)(\bar{L}_L \gamma_\mu \tau^a L_L)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	3.4	3.0	*
$\mathcal{O}_{H1} = (\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)(H^\dagger i D_\mu H)$	$B \to (X)\ell\bar{\ell}, \quad K \to \pi\nu\bar{\nu}, (\pi)\ell\bar{\ell}$	1.6	1.6	*
$\mathcal{O}_{q5} = (\bar{Q}_L \lambda_{\rm FC} \gamma_\mu Q_L)(\bar{D}_R \gamma_\mu D_R)$	$B \to K\pi, \quad \epsilon'/\epsilon, \ldots$	~ 1		

<u>MFV</u> ALLOWS FOR NEW PHYSICS AT THE TEV AND EXPLAINS WHY WE HAVEN'T SEEN IT

<u>MFV</u> GIVES NO CLUE ABOUT WHY MASSES AND MIXINGS ARE WHAT THEY ARE

CAN WE DO BETTER?

1. Consider the Yukawas as true scalar fields that transform under the flavor group

2. Write the scalar potential invariant under flavor

3. Minimize it to find masses and mixings

R. Alonso, B. Gavela, L. Merlo, S. Rigolin; 1103.2915

QUARK CASE

$$\mathscr{L} = \mathscr{L}_{kin} - V(\mathrm{Higgs}) - Y_U \bar{Q} H U_R - Y_D \bar{Q} \tilde{H} D_R + \ldots$$

Flavor Group: $G = SU(3)_Q \times SU(3)_U \times SU(3)_D$

$$Q_L \to V_L Q_L\,, \quad D_R \to V_{dR} D_R\,, \quad U_R \to V_{uR} U_R$$

$$Y_U \to V_L Y_U V_{uR}^\dagger\,, \quad Y_D \to V_L Y_D V_{uR}^\dagger$$

SCALAR POTENTIAL FOR THE YUKAWAS

$$Y_u = \frac{\mathcal{Y}_u}{\Lambda_?}\,, \qquad Y_d = \frac{\mathcal{Y}_d}{\Lambda_?}$$

Not evident what this scale is

FOCUS ON MIXING (for now)

$$\mathrm{Tr}[\mathcal{Y}_u \mathcal{Y}_u^\dagger]\,, \qquad \mathrm{Tr}[\mathcal{Y}_d \mathcal{Y}_d^\dagger]\,, \qquad \mathrm{Tr}[\mathcal{Y}_u \mathcal{Y}_u^\dagger \mathcal{Y}_d \mathcal{Y}_d^\dagger]$$

Only one that contributes to mixing

$$\det[\mathcal{Y}_u]\,, \qquad \det[\mathcal{Y}_d]$$

REARRANGEMENT INEQUALITY

Let $\{a_1, a_2, \cdots, a_n\}$ and $\{b_1, b_2, \cdots, b_n\}$ be lists of positive numbers such that

$$a_1 \geq a_2 \geq \cdots. \geq a_n \quad \text{AND} \quad b_1 \geq b_2 \geq \cdots \geq b_n$$

Then $a_1b_1 + a_2b_2 + \cdots + a_nb_n \geq a_1b_{p(1)} + \cdots + a_nb_{p(n)}$

VON NEUMANN ***TRACE INEQUALITY***

Let A and B be two **matrices** and $\{a_1, a_2, \cdots, a_n\}$ and $\{b_1, b_2, \cdots, b_n\}$ be their eigenvalues with

$a_1 \geq a_2 \geq \cdots. \geq a_n$ AND $b_1 \geq b_2 \geq \cdots \geq b_n$

Then $a_1 b_1 + a_2 b_2 + \cdots + a_n b_n \geq$

$$\mathrm{Tr}\left[\underbrace{y_u y_u^\dagger}_{A} \underbrace{y_d y_d^\dagger}_{B}\right] = \mathrm{Tr}\left[y_u^2 U_{CKM} y_d^2 U_{CKM}^\dagger\right]$$

$$U_{CKM} = \textit{Permutation matrix}$$

In particular

$$U_{CKM} = 1$$

Naive minimization of the Yukawa potential leads to the prediction of no mixing for quarks.

WHAT ABOUT THE LEPTONS?

Neutrinos Majorana or Dirac?

Dirac, same story, let's try

Majorana

SM + 2 FAMILIES OF RH NEUTRINOS

$$\mathscr{L} = \mathscr{L}_{kin} - V(\text{Higgs}) - Y_E \bar{L} H e_R - Y_\nu \bar{L} \tilde{H} N_R - \frac{M_N}{2} \bar{N} N^c + \ldots$$

$$M_N = M \times \mathbb{I}$$

no new source of flavor

$$G = SU(3)_L \times SU(3)_E \times O(2)$$

two families

R. Alonso, B. Gavela, L. Merlo, D.H. ; 1206.3167

only contrib to mixing

$$\mathrm{Tr}\left[Y_E Y_E^\dagger Y_\nu Y_\nu^\dagger\right] = \mathrm{Tr}\left[y_E^2 V y_\nu^2 V^\dagger\right]$$

$V =$ *Permutation matrix*

BUT $V \neq U_{mix}$

$$U_{mix}^T m_\nu U_{mix} = \mathrm{diag}\{m_1,\ m_2\}$$

for $m_\nu \propto Y_\nu Y_\nu^T$

R. Alonso, B. Gavela, L. Merlo, D.H. ; 1206.3167

$$V = \text{Permutation matrix}$$

$$U_{mix} = R(\theta) \cdot \begin{pmatrix} e^{i\alpha} & \\ & e^{-i\alpha} \end{pmatrix}$$

$$\boxed{\alpha = \frac{\pi}{4}, \frac{3\pi}{4}, \qquad \tan 2\theta \propto 2 \sin 2\alpha \frac{\sqrt{m_1 m_2}}{m_1 - m_2}}$$

Majorana phase is determined!

Potentially Large Mixing angles!

WHAT ABOUT THE MASSES?

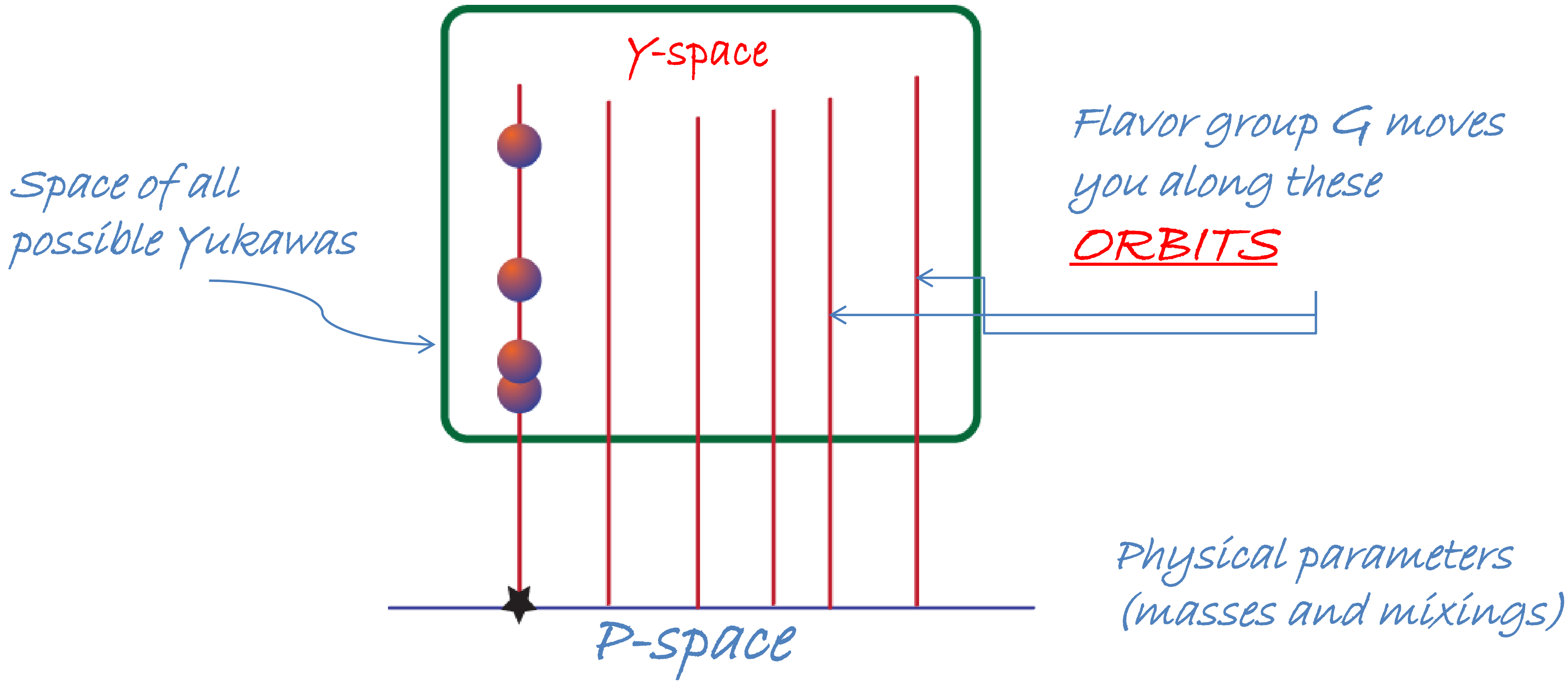
1. Masses seem to depend on many parameters in the potential.
   Just a restatement of the flavor puzzle? ?

2. Neutrino sector flavor structure unclear!

NEED MORE POWERFUL METHOD!

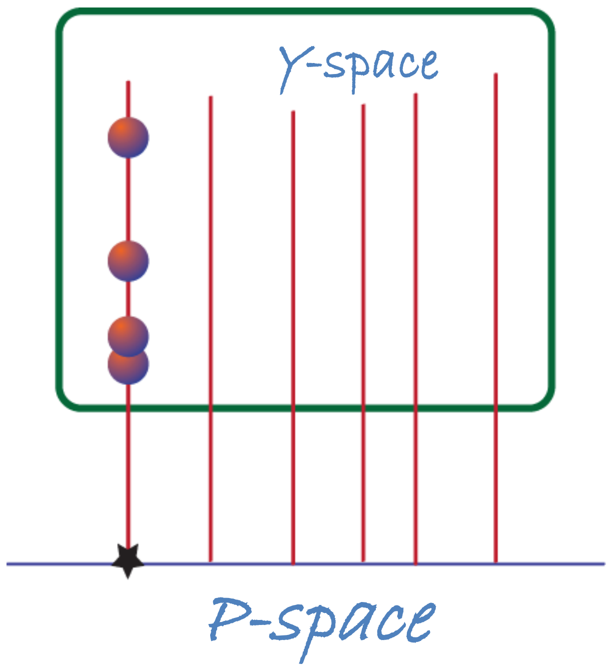
Michel, Radicati, Cabibbo, Maiani

The Geometry of «PHYSICAL PARAMETERS»



Physical parameters are labels for the orbits in *Y*-space with respect to the flavor group *G*

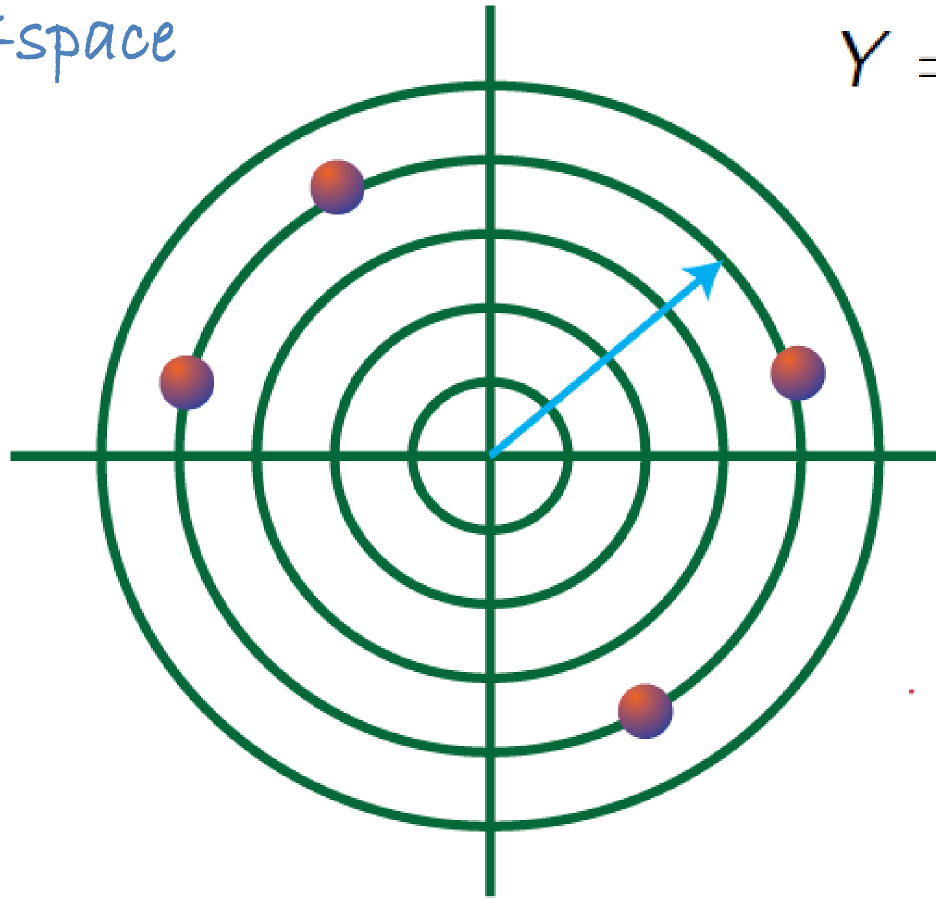
Flavor invariants are functions on Y-space that are constant along the orbits.



Physical parameters are flavor invariants but might not be the most convenient ones.

Toy Y-space

$$Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$



Flavor group

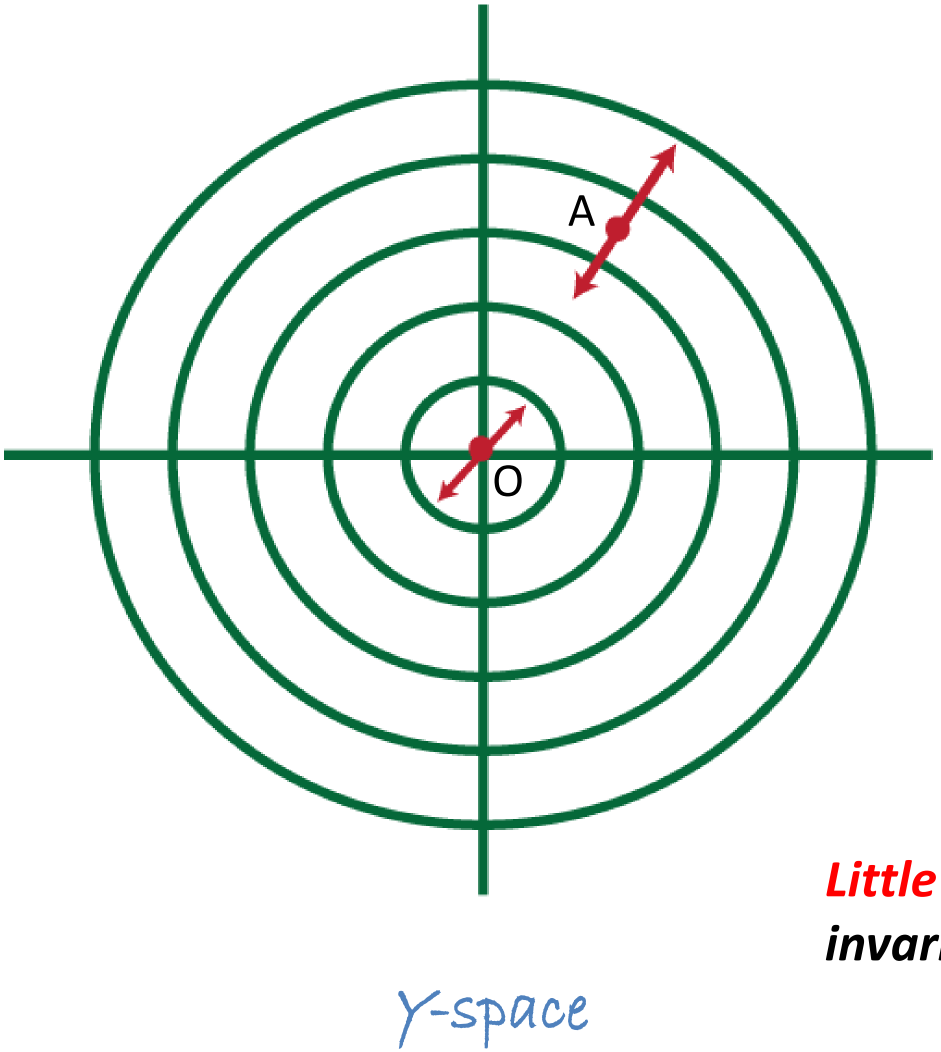
$$G = SO(2)$$

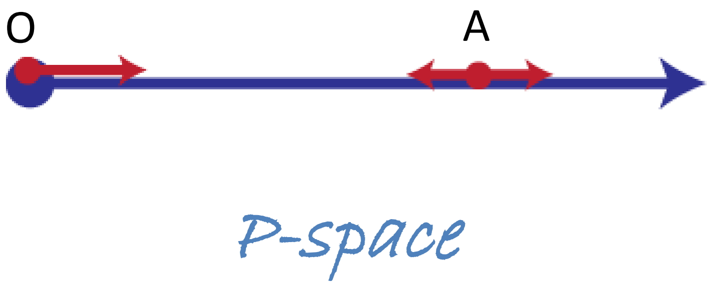
$$P = I(Y) = Y^T Y = y_1^2 + y_2^2 = r^2$$

Boundary

Physical parameter space

SYMMETRY INCREASES AT THE BORDER

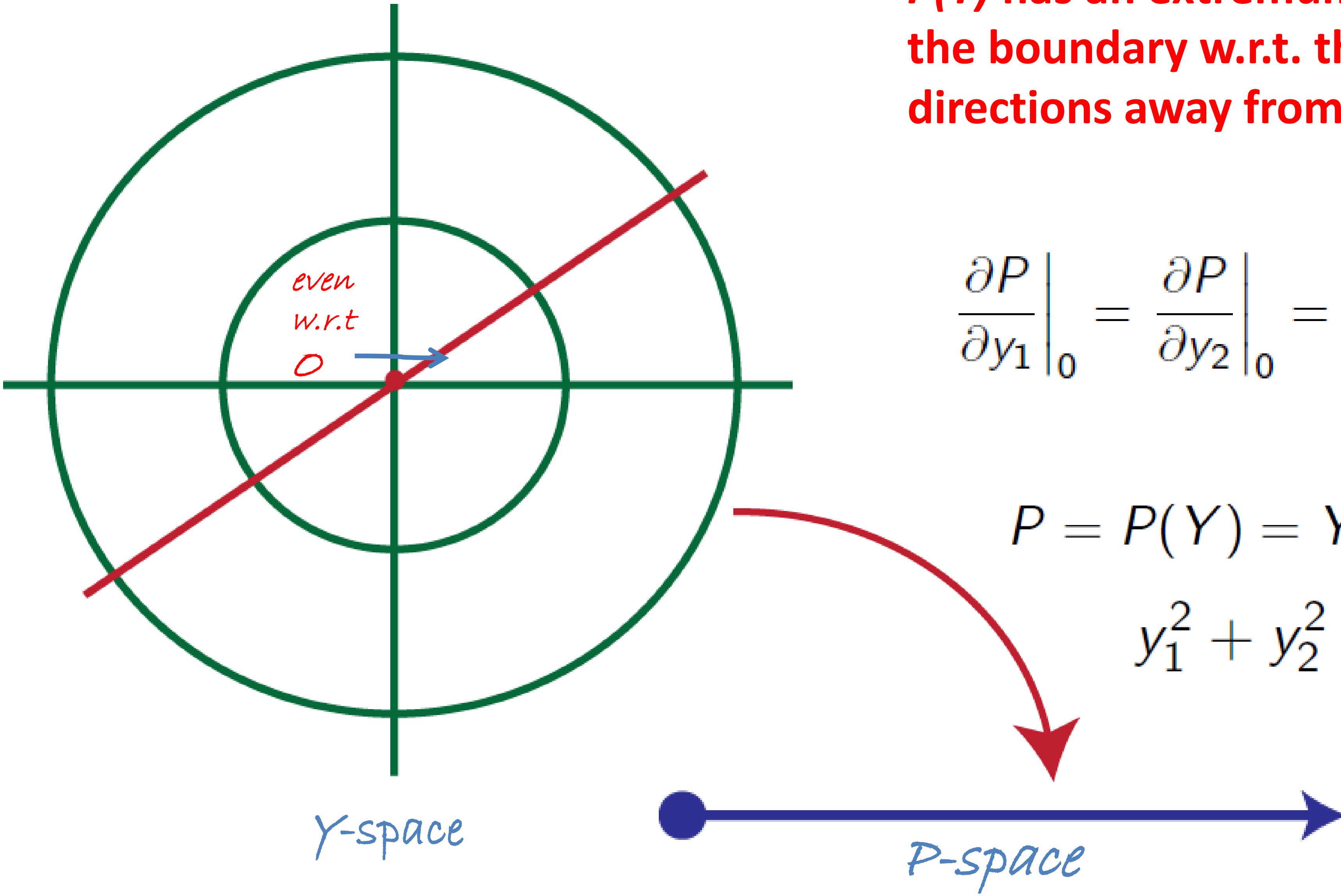




***Little group of point X*: Subgroup that leaves invariant point X in Y-manifold**

P(Y) over a line through the origin

_P(Y)_ has an extremum at the boundary w.r.t. the directions away from it!

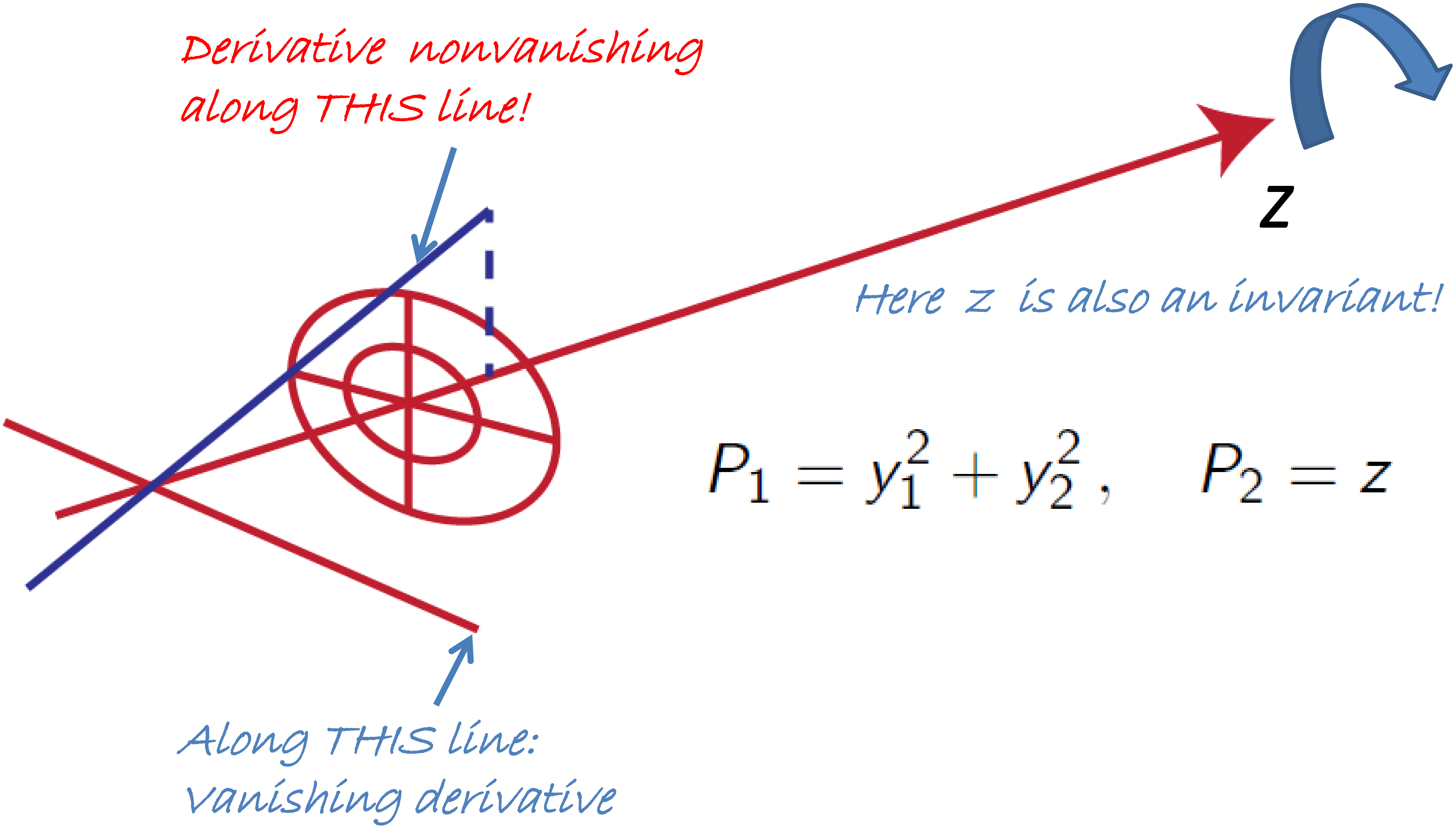


$$\left.\frac{\partial P}{\partial y_1}\right|_0 = \left.\frac{\partial P}{\partial y_2}\right|_0 = 0$$

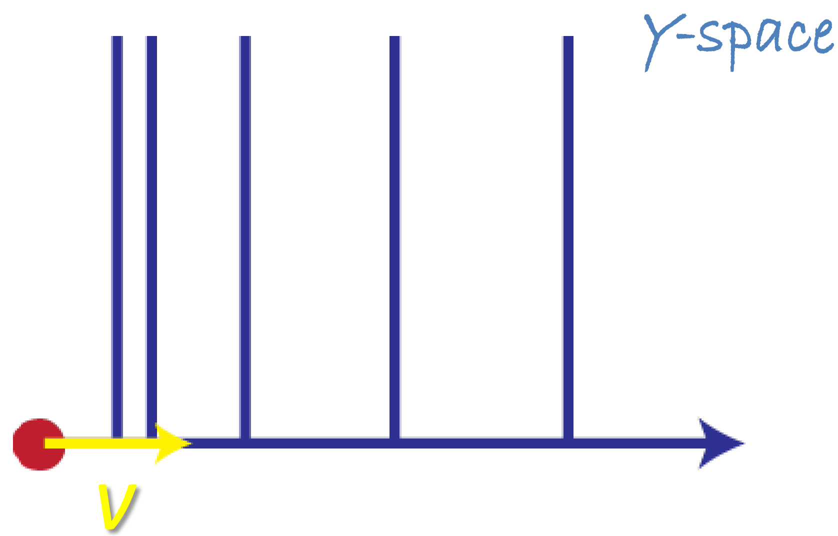
$$P = P(Y) = Y^T Y$$

$$y_1^2 + y_2^2$$

AXIAL SYMMETRY IN 3D

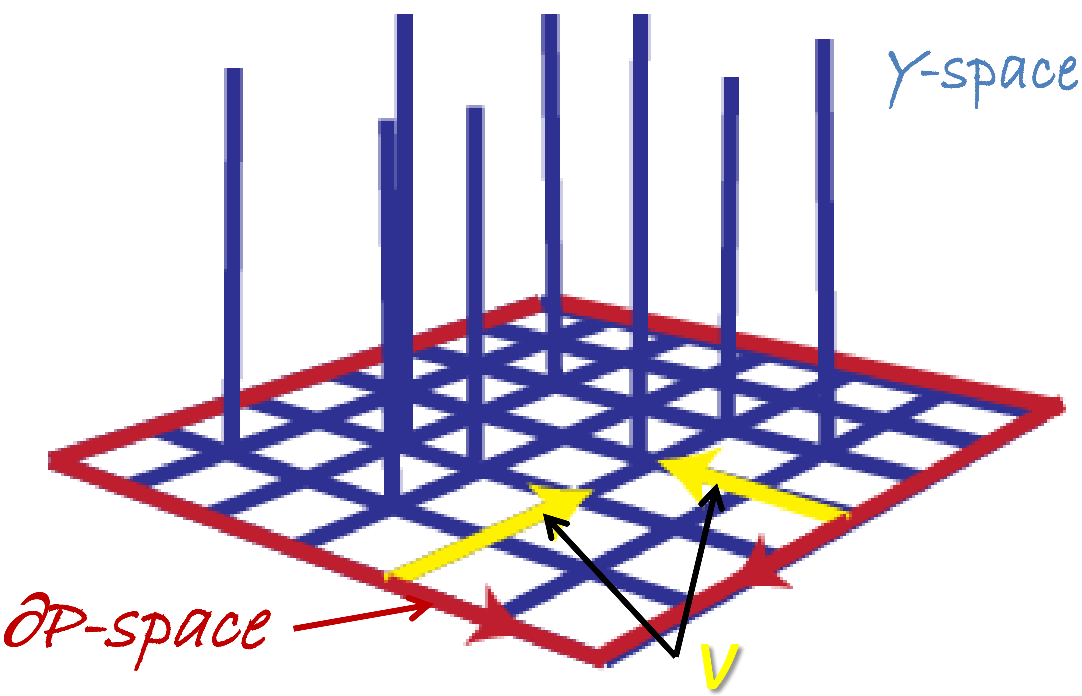


$$P_1 = y_1^2 + y_2^2 \,, \qquad P_2 = z$$



$$\left.\frac{\partial P}{\partial y_1}\right|_0 = \left.\frac{\partial P}{\partial y_2}\right|_0 = 0$$

$$\vec{v} = (1,\, 0) \quad or \quad (0,\, 1)$$



For all invariants

$$\forall i\,, \quad \vec{v}\cdot\frac{\partial P_i}{\partial \vec{y}} = 0$$

at the points of the boundary

THE SCALAR POTENTIAL

$$\frac{\partial V(\vec{y})}{\partial \vec{y}} = \sum_i \frac{\partial V(\vec{y})}{\partial P_i} \frac{\partial P_i}{\partial \vec{y}}$$

$$\Downarrow$$

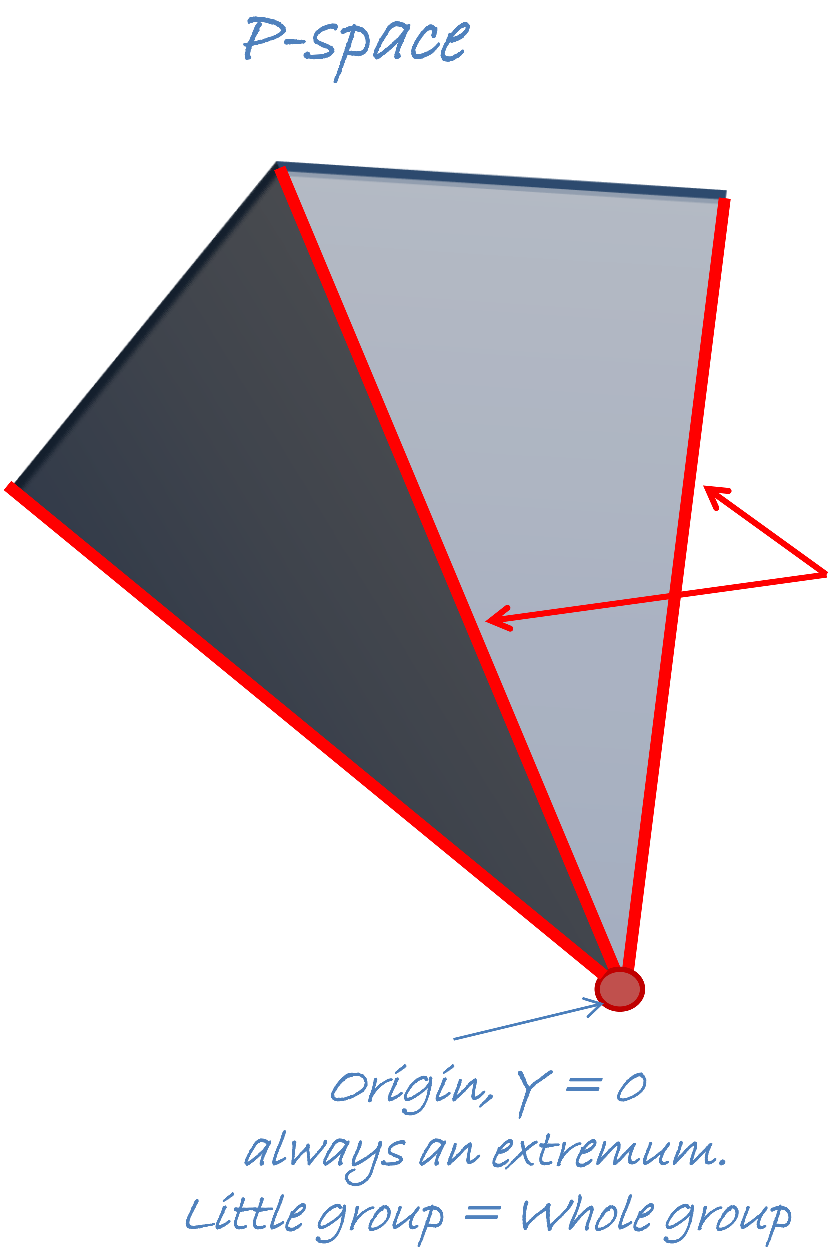
$$\vec{v} \cdot \frac{\partial V(\vec{y})}{\partial \vec{y}} = \vec{v} \sum_i \frac{\partial V(\vec{y})}{\partial P_i} \frac{\partial P_i}{\partial \vec{y}} = 0$$

$$\Downarrow$$

The potential at the boundary has vanishing directional derivatives towards the «insides» of Y-space

SO FAR…

1. Space of physical parameters has boundaries where the symmetry increases
2. Derivatives of the invariants along directions away from the boundary vanish
3. Extrema w.r.t. to the boundary of the invariant space are extrema of the full Y-space.



Stratum: Set of points in P-space with the same little group

HOW TO MINIMIZE THE POTENTIAL?

1. Determine the boundaries of *P-space*

2. Starting from the origin, minimize along strata of maximal symmetry. (The larger the remaining symmetry, the more directional derivatives that vanish automatically)

3. Establish correspondence with masses and mixings.

R. Alonso, B. Gavela, G. Isidori, L. Maiani, E. Nardi, C. S. Fong

QUARK CASE

$$Y_U \to U_q Y_U U_U^\dagger \,, \qquad Y_D \to U_q Y_D U_D^\dagger$$

INVARIANTS

$$P_{U1} = \mathrm{Tr}\left[Y_U Y_U^\dagger\right] \,, \quad P_{U2} = \mathrm{Tr}\left[(Y_U Y_U^\dagger)^2\right] \,, \quad P_{U3} = \mathrm{Tr}\left[(Y_U Y_U^\dagger)^3\right]$$

$$P_{D1} = \mathrm{Tr}\left[Y_D Y_D^\dagger\right] \,, \quad P_{D2} = \mathrm{Tr}\left[(Y_D Y_D^\dagger)^2\right] \,, \quad P_{D3} = \mathrm{Tr}\left[(Y_D Y_D^\dagger)^3\right]$$

$$P_{UD} = \mathrm{Tr}\left[Y_U Y_U^\dagger Y_D Y_D^\dagger\right] \,, \quad P_{U2D} = \mathrm{Tr}\left[(Y_U Y_U^\dagger)^2 Y_D Y_D^\dagger\right]$$

$$P_{UD2} = \mathrm{Tr}\left[Y_U Y_U^\dagger (Y_D Y_D^\dagger)^2\right] \,, \quad P_{2UD} = \mathrm{Tr}\left[(Y_U Y_U^\dagger Y_D Y_D^\dagger)^2\right]$$

PHYSICAL PARAMETERS

$$U_{CKM} = \begin{pmatrix} c\theta & s\theta & \\ -s\theta & c\theta & \\ & & 1 \end{pmatrix}$$

+

$$M_q = \mathrm{diag}\{0,\ 0,\ m\} \to SU(2) \times SU(2) \times U(1)$$

or $M_q = \mathrm{diag}\{m,\ m,\ m\} \to SU(3)$

Symmetry at the boundary strata

LEPTON CASE

$$G = U(3)_L \times U(3)_E \times SO(3)_N$$ Majorana!

$$G = U(3)_L \times U(3)_E \times SO(2)_N$$

For the charged leptons, same thing as for the quarks

$$Y_E = \mathrm{diag}\{0,\ 0,\ y\}\,, \quad U(3)_L \times U(3)_E \to U(2)_L \times U(2)_E \times U(1)$$

$$Y_E = \mathrm{diag}\{y,\ y,\ y\}\,, \quad U(3)_L \times U(3)_E \to U(3)_{vec}$$

New invariants in the neutrino sector

$$\mathrm{Tr}\left[Y_\nu^\dagger Y_\nu Y_\nu^T Y_\nu^*\right]\,,\quad \mathrm{Tr}\left[(Y_\nu^\dagger Y_\nu)^2 Y_\nu^T Y_\nu^*\right]$$

$$\Downarrow$$

$$m_\nu = \mathrm{diag}\{m,\ m,\ m\}\,,\quad U(3)_L \times O(3) \to O(3)_{vec}$$

$$m_\nu = \mathrm{diag}\{m,\ m,\ m'\}\,,\quad U(3)_L \times SO(2)_N \to U(1)$$

One maximal mixing angle!

R. Alonso, B. Gavela, L. Merlo, S. Rigolin, D.H. ; 1306.5927

BEST SCENARIO

$$m_\nu = \mathrm{diag}\{m,\ m,\ m'\}\,, \qquad U(3)_L \times SO(2)_N \to U(1)$$

$$Y_E = \mathrm{diag}\{0,\ 0,\ y\}\ +\ m_\nu\,, \qquad G \to SU(2)_E \times U(1)$$

$$\alpha = \pi/4\,, \quad \theta_{23} \sim \frac{\pi}{4}\,, \quad \theta_{13} = 0$$

Minimization of the potential leads to phenomenologically relevant Yukawas fof both the leptons and the quarks.

WHERE DOES IT FAIL?

- **Global symmetry breaking -> Goldstones**
- **The Return of the Hierarchy**
- **Beyond leading order??**

BEYOND LEADING ORDER?

Lepton case, not so difficult

$$m_\nu = \mathrm{diag}\{m', m, m\}\,, \quad U(3)_L \times SO(2)_N \to U(1)$$

Quark case

$$M_q = \mathrm{diag}\{0, 0, m\}$$

Simple

$$M_q = m \cdot \mathrm{diag}\{\epsilon_1, \epsilon_2, 1\}$$

HARD!

BEYOND LEADING ORDER?

Can we get $M_q = m \cdot \mathrm{diag}\{\epsilon_1, \epsilon_2, 1\}$ *?*

- **Quantum corrections?** **Doesn't work**

Eff. potential $$V(Y) = V_0(Y) + \epsilon V_1(Y) + \ldots$$

$$M^2 = M_0^2 + \epsilon M_1^2 \,, \quad M^2 = \frac{\partial V}{\partial Y_i \partial Y_j}$$

J. Espinosa, E. Nardi, C. S. Fong; 1211.6428
H. Georgi, A. Pais; Phys. Rev. D16 (1977) 3520

BEYOND LEADING ORDER?

- **Nonrenormalizable terms?**

Same argument…

$$M_q = m \cdot \mathrm{diag}\{\epsilon_1,\, \epsilon_2,\, 1\}$$

MUST APPEAR AT TREE LEVEL!

BEYOND LEADING ORDER?

- **Extended field content?**

This works but reintroduces the flavor puzzle. All the complicacy goes to the scalar potential. No preference for «small perturbation» solution

Maybe different symmetry breaking technology needed

J. Espinosa, E. Nardi, C. S. Fong; 1211.6428

GOLDSTONES? OK, GAUGE

ANOMALIES?

OK, MAKE VECTOR-LIKE

THE HIERARCHY PROBLEM

Usually understood as:

$$\delta m_h^2 = \frac{\Lambda^2}{16\pi^2}\left(6\lambda + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6y_t^2\right) + \ldots$$

NOT A PROBLEM IN THIS TALK!

Hierarchy Problem: ***Large, finite quantum corrections to the Higgs mass proportional to the scale of new physics.***

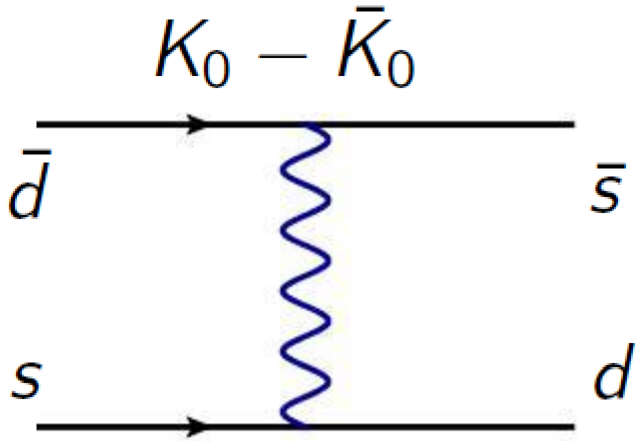
HIERARCHY AFFLICTED THEORIES

- *Grand Unified Theories*
- *Neutrino masses. Seesaw*
- *Axions*
- *……*

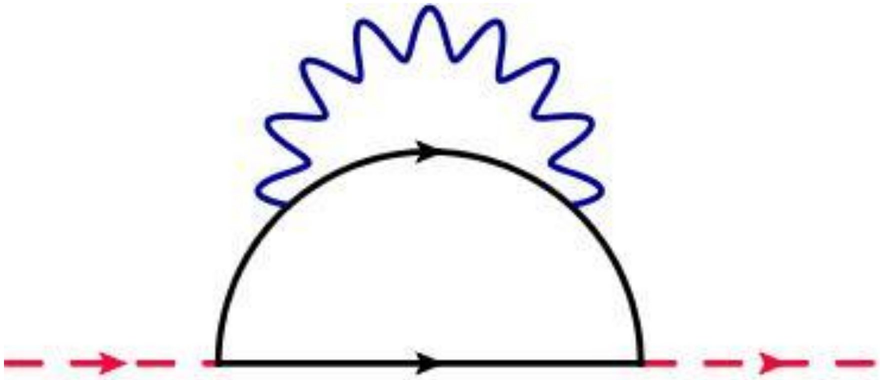
DEALING WITH IT

- *Forget about it*
- *Assume your new physics is around the TeV or lower (SUSY, Little Higgs...)*
- *Come up with a new solution if you can.*

H.P. IN GAUGE FLAVOR SYMMETRIES



$$\frac{M_X}{g_{fl}} \sim 10^{3-4}\ \mathrm{TeV}$$



$$\sim \frac{1}{16\pi^2} y_t g^2 \cdot \frac{M_X}{g}$$

$$g_{fl} \sim 10^{-1} - 10^{-2}$$

NOT SO BAD

H.P. IN GAUGE FLAVOR SYMMETRIES

Yukawas give mass to the gauge bosons

$$\langle \mathcal{Y} \rangle \sim \frac{M_{fl}}{g_{fl}} \sim 10^{3-4}\ \mathrm{TeV}$$

But for the quark masses we have

$$\frac{\langle \mathcal{Y} \rangle}{\Lambda_?} Q_L H U_R \quad \Longrightarrow \quad \Lambda_? \sim \langle \mathcal{Y} \rangle \frac{m_{top}}{m_{up}}$$

Hard to avoid $m_H \propto \Lambda_?$

H.P. IN GAUGE FLAVOR SYMMETRIES

~~*Assumption: There exists*~~ $\cancel{\Lambda_? \gg M_{fl}}$

Assumption: Effective Yukawas are suppressed by the flavor breaking scale

$$\frac{\Lambda_?}{\langle \mathcal{Y} \rangle} Q_L H U_R$$

Notice this is just a «Seesaw»

$$\frac{c}{M_{LN}} LLHH$$

Lepton Number violating scale

$$\frac{\Lambda_?}{\langle\mathcal{Y}\rangle} Q_L H U_R$$

Eigenvalues of $\langle\mathcal{Y}\rangle \propto \left\{\frac{1}{m_u}, \frac{1}{m_c}, \frac{1}{m_t}\right\}$

$$\langle\mathcal{Y}\rangle \sim \Lambda_{fl} \longleftarrow$$

Scale of Flavor Violation

From FCNCs and flavor SB $\Lambda_{fl} \gtrsim 10^4\,\mathrm{TeV}$

$$m_u \sim \frac{v_H \Lambda_?}{\Lambda_{fl}} \Rightarrow \frac{\Lambda_?}{v_h} \sim 1 - 10^{-1}$$

$$\frac{\Lambda_?}{\langle Y \rangle} Q_L H U_R \quad \textit{with} \quad \frac{\Lambda_?}{v_h} \sim 1 - 10^{-1}$$

This might be too low!

Precise model building in progress…

B. Grinstein, M. Redi, G. Villadoro; 1009.2049
J. Berryman, D. H. (work in progress)

- *MFV violation, still good, lowers flavor scale to almost no tension with Hierarchy. But smells of defeat.*

- *Yukawas as scalar fields. A step forward? Promising zeroth order vevs almost forced on you. Both for quarks and leptons.*

- *Going beyond is tricky. Reintroduction of the flavor problem? Reintroduction of the Hierarchy? Some new ideas seem to be required.*

- *A couple of cute mathematical techniques.*

- ***How to get the SM spectrum and mixings in all their complicacies? New ideas to lift the zero eigenvalues?***

THE DISCRETE PREDICTIONS

D. H. and A. Yu. Smirnov;
1204.0445, 1212.2149 ,
1304.7738

A theory of flavor

A theory of masses*: One that predicts a structure for Y and M independently without saying anything about the other*

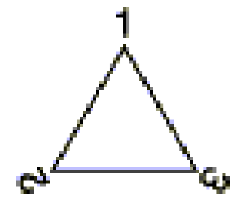
A theory of mixing*: One that predicts the misalignment between Y and M without necessarily speaking about the eigenvalues of each.*

DISCRETE SYMMETRIES form part of such a theory of flavor

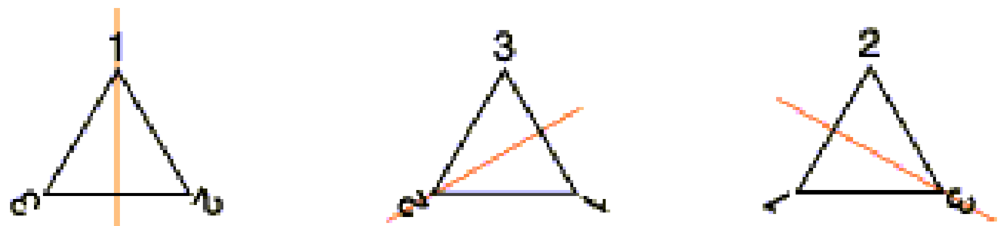
WHAT ARE (NONABELIAN) DISCRETE SYMMETRIES?

* Images taken from http://www.olympus.net/personal/mortenson/preview/definitionss/symmetrytransform.html

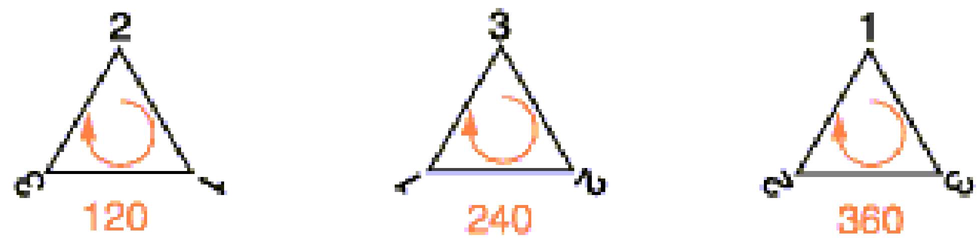
Generators



Reflections (P)



120° Rotations (O)



Do not commute

$$PO \neq OP$$

Raised to some power, equal the identity

FOR INSTANCE $P^2 = O^3 = 1$

A4: symmetry group of the tetrahedron

S4:Full permutation symmetry of 4 elements

A5: Symmetry group of the icosahedron

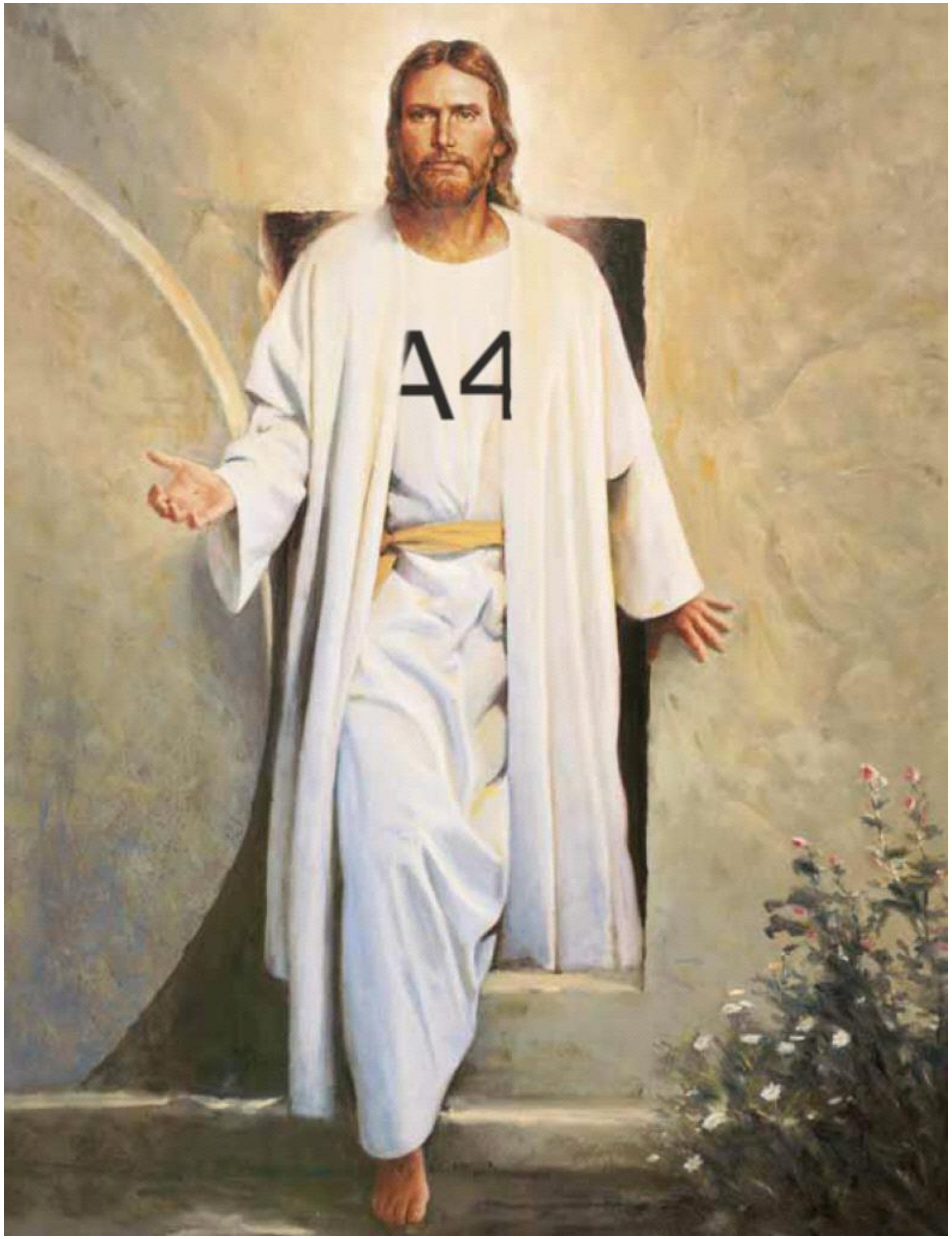
…………………………………………………………………

Important fact: ***These symmetries have 3-dimensional representations that account for the presence of 3 families***

$$|U_{TBM}|^2 = \begin{array}{c c} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{pmatrix} 2/3 & 1/3 & 0 \\ 1/6 & 1/3 & 1/2 \\ 1/6 & 1/3 & 1/2 \end{pmatrix} \end{array}$$

$$|U_{BM}|^2 = \begin{array}{c c} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{pmatrix} \end{array}$$

- *Weren't discrete symetries DEAD?*
- *That is, didn't they predict $\theta_{13} = 0$?*



$\theta_{13} = 0$ IS ***NOT*** A PREDICTION OF DISCRETE SYMMETRIES

Flavor Symmetry

$$G_{fl}$$

symmetry breaking

Charged Leptons

$$G_\ell$$

symmetry breaking

Neutrinos

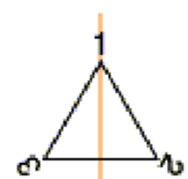
$$G_\nu$$

Residual symmetries

Example: G_{fl} ***the symmetry of the equilateral triangle***

$$G_\ell \equiv O$$

$$G_\nu \equiv P$$



BOTTOM UP

1. *Find accidental symmetries of the charged lepton and neutrino mass terms*

2. *Choose discrete subgroups in both cases*

3. *Combine them to define G_{fl}*

IN DETAIL

1.- Identify the accidental symmetries

$$\mathscr{L} = \frac{g}{\sqrt{2}} \bar{\ell}_L U_{PMNS} \gamma^\mu \nu_L W_\mu^+ + \bar{E}_R m_\ell \ell_L + \frac{1}{2} \bar{\nu^c}_L m_\nu \nu_L + \ldots + \text{h.c.}$$

Charged Leptons

$\bar{E}_R m_\ell \ell_L$ *is invariant under* $U(1)^3$ ***accidental***

$$E_R \to T E_R\,, \quad \ell_L \to T \ell_L \qquad T = \text{diag}\{e^{i\alpha}, e^{i\beta}, e^{i\gamma}\}$$

IN DETAIL

1.- Identify the accidental symmetries

$$\mathscr{L} = \frac{g}{\sqrt{2}}\bar{\ell}_L U_{PMNS}\gamma^\mu \nu_L W^+_\mu + \bar{E}_R m_\ell \ell_L + \frac{1}{2}\bar{\nu^c}_L m_\nu \nu_L + \ldots + \text{h.c.}$$

Neutrinos

$\frac{1}{2}\bar{\nu^c}_L m_\nu \nu_L$ *invariant under* $Z_2 \otimes Z_2$ *accidental*

$$S_1 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix}, \quad S_2 = \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix}, \quad S_3 = S_1 S_2 = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix}$$

$$\mathscr{L} = \frac{g}{\sqrt{2}}\bar{\ell}_L U_{PMNS}\gamma^\mu \nu_L W^+_\mu + \bar{E}_R m_\ell \ell_L + \frac{1}{2}\bar{\nu^c}_L m_\nu \nu_L + \ldots + \text{h.c.}$$

$$M_\nu = U^* m_\nu U^\dagger$$

$$M_\ell = m_\ell V$$

$$U_{PMNS} = VU$$

Change of basis

$$\mathscr{L} = \frac{g}{\sqrt{2}}\bar{\ell}_L \gamma^\mu \nu_L W^+_\mu + \bar{E}_R M_\ell \ell_L + \frac{1}{2}\bar{\nu^c}_L M_\nu \nu_L + \cdots + \text{h.c.}$$

Take $U \equiv U_{PMNS} \quad V \equiv 1$

Invariance of M_ν *under* $Z_2 \otimes Z_2$ *accidental* $\Rightarrow$ $S^\dagger_{iU} M_\nu S_{iU} = M_\nu$ *with* $S_{iU} = U S_i U^\dagger$

Still $S^2_{iU} = 1$

2.- Choose the flavor subgroups

For the neutrinos

Simply choose at least one of the S_{iU}

2.- Choosing the flavor subgroups

For charged leptons, use a ***finite abelian*** *subgroup of $U(1)^3$ as the group of flavor*

Impose $T^m = 1$, T *unitary*

$$T = \begin{pmatrix} e^{2\pi i k_1/m} & & \\ & e^{2\pi i k_2/m} & \\ & & e^{-2\pi i (k_1+k_2)/m} \end{pmatrix}$$

3.- Define the flavor group

- *Define a relation between* S_{iU} *and* T

We had $T^m = 1, \quad S_{iU}^2 = 1$

Add $(S_{iU}T)^p = (US_iU^\dagger T)^p = \mathbb{I}$

The relations

$$S_{iU}^{n} = T^{m} = (S_{iU}T)^{p} = \mathbb{I}$$

define the ***von Dyck group*** $D(n, m, p)$

$D(2, 2, p)$ *is the dihedral group* $\mathbf{D}_p$

$$D(2,2,3) = \mathbf{S}_3$$
$$D(2,3,3) = \mathbf{A}_4$$
$$D(2,3,4) = \mathbf{S}_4$$
$$D(2,3,5) = \mathbf{A}_5$$

Notice that if

$$\frac{1}{n} + \frac{1}{m} + \frac{1}{p} \leq 1$$

The von Dyck group is infinite

$$S_{iU}^{n} = T^{m} = (S_{iU}T)^{p} = \mathbb{I}$$

$$\frac{\pi}{n} + \frac{\pi}{m} + \frac{\pi}{p} > \pi$$

Positive curvature space

$$\frac{\pi}{n} + \frac{\pi}{m} + \frac{\pi}{p} = \pi$$

Flat space

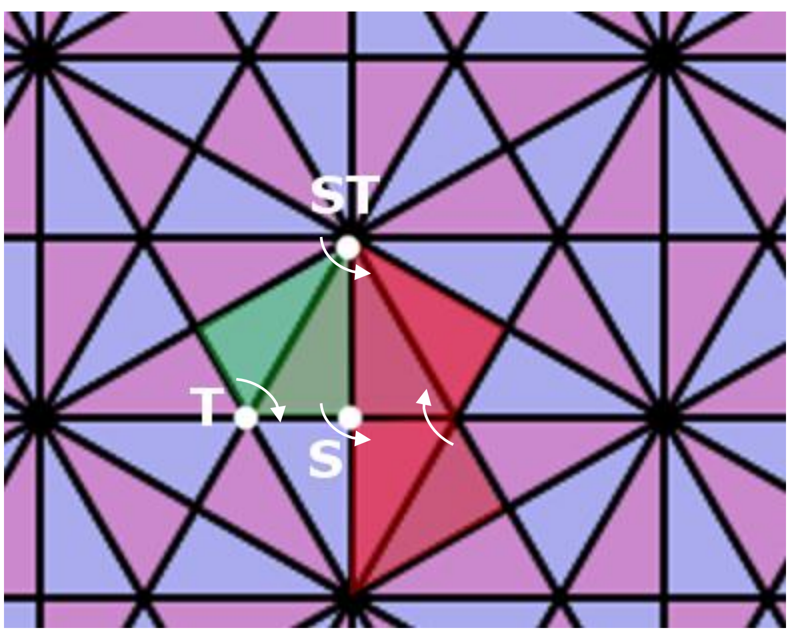
$$\frac{\pi}{n} + \frac{\pi}{m} + \frac{\pi}{p} < \pi$$

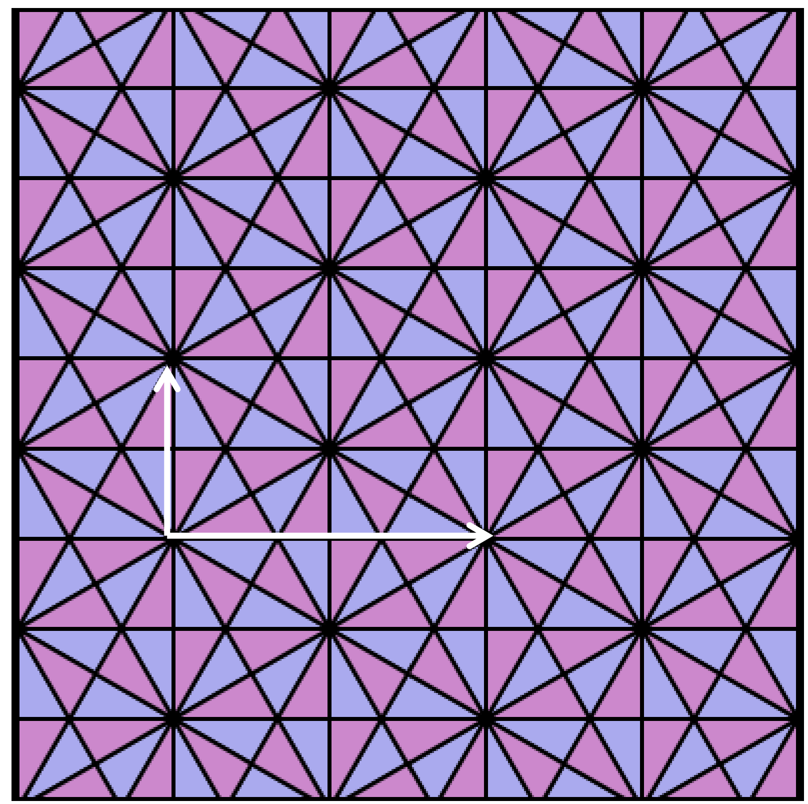
Negative curvature space

Take $n = 2\,, \quad m = 3\,, \quad p = 6$

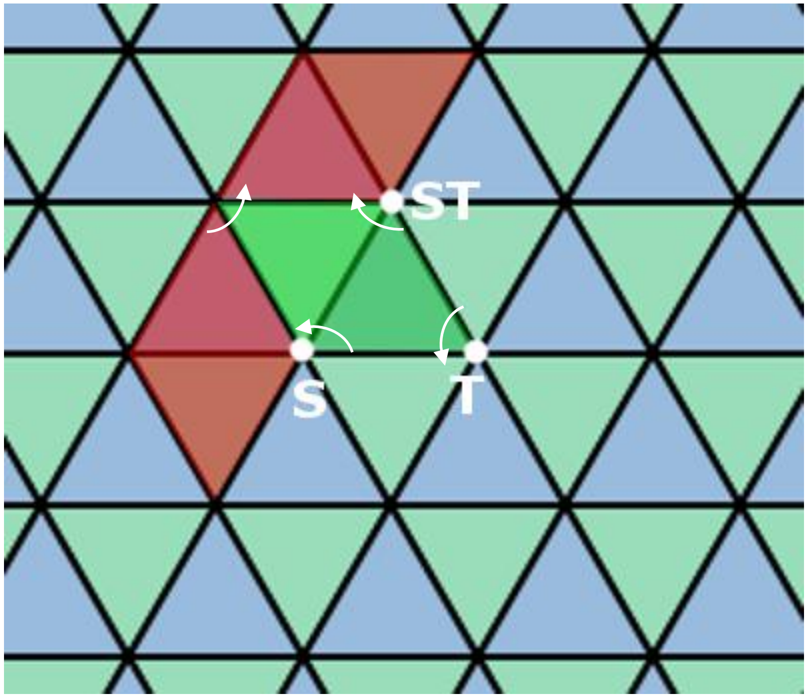
$$\frac{\pi}{2} + \frac{\pi}{3} + \frac{\pi}{6} = \pi$$

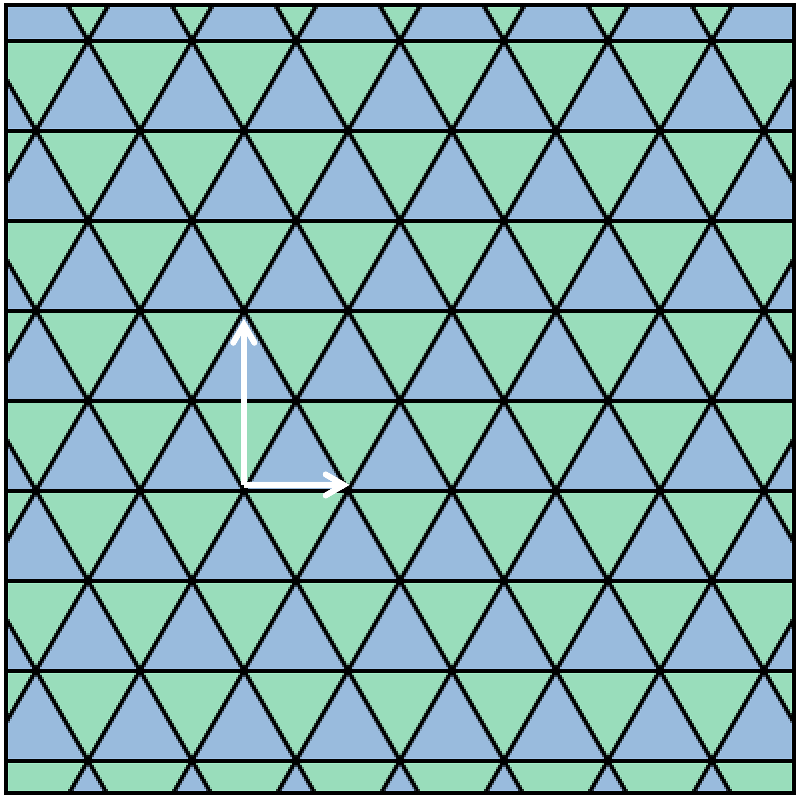
$$S^2 = T^3 = (ST)^6 = 1$$





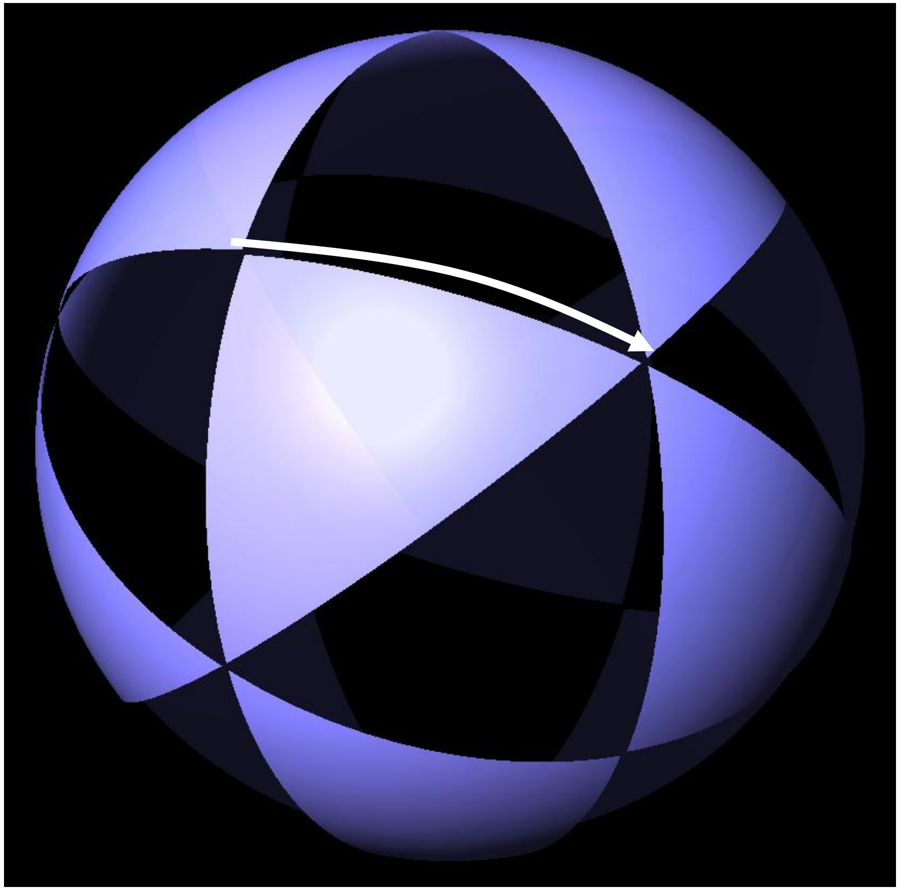
$$n = 3\,, \quad m = 3\,, \quad p = 3$$





$$S_{iU}^n = T^m = (S_{iU}T)^p = \mathbb{I}$$

$$\frac{\pi}{n} + \frac{\pi}{m} + \frac{\pi}{p} > \pi$$ *corresponds to tesselations of the sphere*



Finite number of translations!

Now that we know the flavor group and the symmetry breaking pattern, find constraints on mixing.

CONSTRAINTS ON THE MIXING MATRIX

$$W_i = S_{iU} T = U S_i U^\dagger T \, , \quad W_i^p = 1$$

$$\mathrm{Det}[W_i - \lambda \mathbb{I}] = 0$$ ***cubic equation with*** $$\lambda_i^p = 1$$

$$\lambda^3 + a\lambda^2 - a^*\lambda - 1 = 0$$ with $$a = -\mathrm{Tr}[W_i]$$

Two equations, one for the real and one for the imaginary part of a

TWO CONSTRAINTS ON THE MIXING MATRIX

$$W_i = S_{iU} T = U S_i U^\dagger T\,, \quad W_i^p = 1$$

The constraints on the entries of the mixing matrix depend on

$$a = -\mathrm{Tr}[W_i]$$

$$T = \begin{pmatrix} e^{2\pi i k_1/m} & & \\ & e^{2\pi i k_2/m} & \\ & & e^{-2\pi i (k_1+k_2)/m} \end{pmatrix}$$

and which S_i is chosen

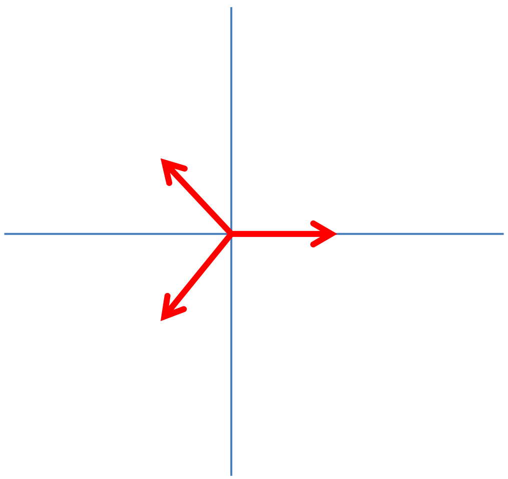
$$S_1 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix} \quad S_2 = \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \quad S_3 = S_1 S_2 = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix}$$

$$W_i = S_{iU}T = US_iU^\dagger T\,, \quad W_i^p = 1$$

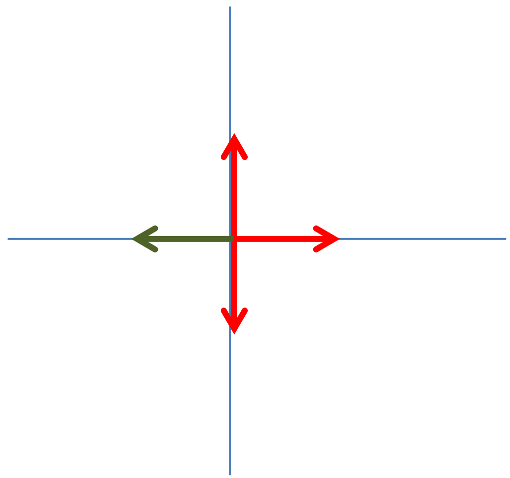
Constraints on the mixing matrix

$$a = -\mathrm{Tr}[W_i]$$

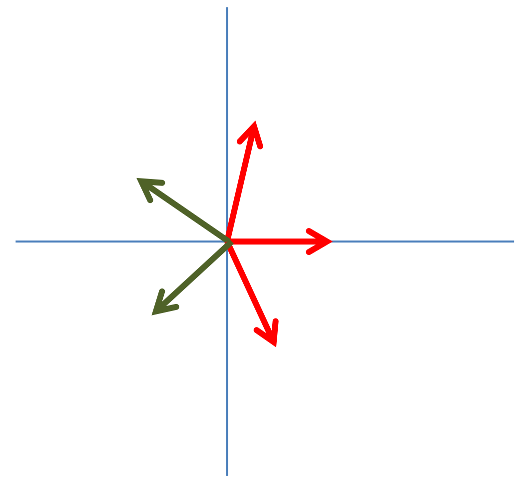
p=3



p=4



p=5



Example: $p = 3 \longrightarrow (\lambda - 1)(\lambda - \omega)(\lambda - \omega^2) = \lambda^3 - 1 \longrightarrow a = 0$

or $p = 4 \longrightarrow (\lambda - 1)(\lambda + i)(\lambda - i) = \lambda^3 - \lambda^2 + \lambda - 1 \longrightarrow a = -1$

The moduli squared of one column are determined (two constraints plus unitarity)

$$
|U_{l\nu}|^2 = \begin{array}{ccc} S_1 & S_2 & S_3 \\ \multicolumn{3}{c}{\begin{pmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu 1}|^2 & |U_{\mu 2}|^2 & |U_{\mu 3}|^2 \\ |U_{\tau 1}|^2 & |U_{\tau 3}|^2 & |U_{\tau 3}|^2 \end{pmatrix}} \end{array}
$$

$$R_i = \mathrm{Re}\{\mathrm{Tr}[W_i + T]\}$$

$$I_i = \mathrm{Im}\{\mathrm{Tr}[W_i + T]\}$$

$$|U_{ei}|^2 = -\frac{R_i \cos\left(\pi \frac{k_1}{m}\right) - 2\cos\left(\pi \frac{k_1+2k_2}{m}\right) - I_i \sin\left(\pi \frac{k_1}{m}\right)}{4 \sin\left(\pi \frac{k_1-k_2}{m}\right) \sin\left(\pi \frac{2k_1+k_2}{m}\right)}$$

$$|U_{\mu i}|^2 = \frac{R_i \cos\left(\pi \frac{k_2}{m}\right) - 2\cos\left(\pi \frac{2k_1+k_2}{m}\right) - I_i \sin\left(\pi \frac{k_2}{m}\right)}{4 \sin\left(\pi \frac{k_1-k_2}{m}\right) \sin\left(\pi \frac{k_1+2k_2}{m}\right)}$$

$$|U_{\tau i}|^2 = -\frac{R_i \cos\left(\pi \frac{k_1+k_2}{m}\right) - 2\cos\left(\pi \frac{k_1-k_2}{m}\right) + I_i \sin\left(\pi \frac{k_1+k_2}{m}\right)}{4 \sin\left(\pi \frac{2k_1+k_2}{m}\right) \sin\left(\pi \frac{k_1+2k_2}{m}\right)}$$

A particular case for T (lazy' case)

$$a = -\mathrm{Tr}[W_i]$$

$$T_e = \begin{pmatrix} 1 & & \\ & e^{2\pi i k/m} & \\ & & e^{-2\pi i k/m} \end{pmatrix} \quad T_\mu = \begin{pmatrix} e^{2\pi i k/m} & & \\ & 1 & \\ & & e^{-2\pi i k/m} \end{pmatrix} \quad T_\tau = \begin{pmatrix} e^{2\pi i k/m} & & \\ & e^{-2\pi i k/m} & \\ & & 1 \end{pmatrix}$$

$$|U_{\beta i}|^2 = |U_{\gamma i}|^2$$
$$|U_{\alpha i}|^2 = \eta, \qquad \beta, \gamma \neq \alpha$$

$$\eta \equiv \frac{1-a}{4\sin^2\left(\frac{\pi k}{m}\right)}$$

Remember

$$|U_{l\nu}|^2 = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{pmatrix} \sim 0.7 & \sim 0.3 & \sim 0 \\ \sim 0.1 & \sim 0.5 & \sim 0.4 \\ \sim 0.2 & \sim 0.2 & \sim 0.6 \end{pmatrix} \end{array}$$

Hence in this case, either $i = 2$ ***or*** $\alpha = e$

Actually, we have shown that the lazy case is unavoidable if the von Dyck group is finite!

***Recapitulating**: What I have shown (under some - mostly harmless - assumptions)*

After a number of choices have been made

1. *The **T-charge** of the charged leptons (k_1 and k_2 value)*
2. *The **order of T** (m value)*
3. *The **S-charges** of the neutrinos*
4. *The **eigenvalues of ST** (a value)*

A two-dimensional surface is cut in the parameter space of the mixing matrix.

THIS IS ALL DISCRETE SYMMETRIES CAN TELL YOU FOR SURE ABOUT MIXING!

Is it possible to fit the measured values of the PMNS matrix??

Choose $\alpha = e$ *in the 'lazy' case*

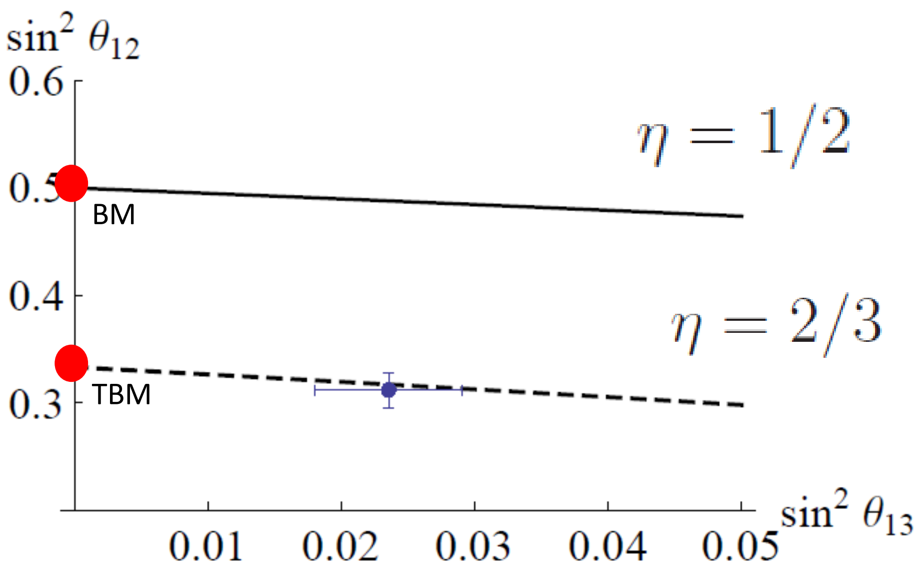
$$S_{iU}^n = T^m = (S_{iU}T)^p = \mathbb{I}$$

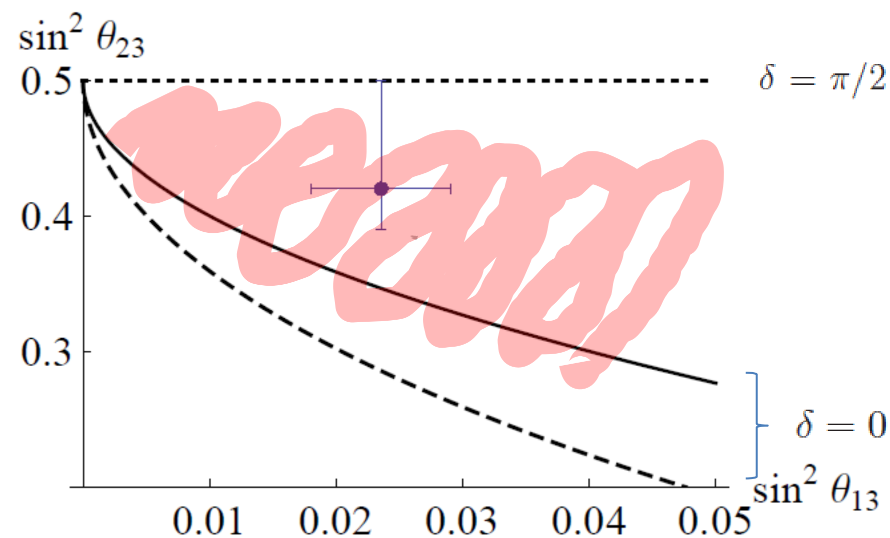
$$\lambda^3 + a\lambda^2 - a^*\lambda - 1 = 0$$

$$\eta \equiv \frac{1-a}{4\sin^2\left(\frac{\pi k}{m}\right)}$$

Taking $i = 1$

- *Solid: m = 4, p = 3. k=1 and from* $(\lambda-1)(\lambda-\omega)(\lambda-\omega^2) = \lambda^3 - 1$ *, a=0 . Group is* $\boldsymbol{S_4}$
- *Dashed: m = 3, p = 4. k=1, a=-1. Group is* $\boldsymbol{S_4}$





CONCLUSIONS (Discrete)

- *Constraints on mixing from discrete symmetries are model independent…*
- *… and can be obtained in a systematic, rather simple way*
- *θ_{13} is not forced to be zero. In general, two parameters are predicted out of 4 in the mixing matrix.*
- *Many results can be readily obtained: TBM, BM and analysis of less known groups made easy.*

CONCLUSIONS (Discrete)

- *IF a larger residual symmetry is imposed in the neutrino sector, up to 4 constraints in the mixing matrix.*
- *In this case, there's still compatibility with measured mixings. Masses predicted degenerate.*